Unit 4 Rational Expressions

Factoring Review

To factor means to break something into smaller parts
ex 6 = 2 x 3

With polynomials, factoring consists of 4 different basic techniques.

I. Common Factors

- Look for the largest term that comes from each part of the expression.

ex: \[ 12x + 8 = 4(3x + 2) \]
GCF = 4
\[ 12 \div 4 = 3 \]
\[ 8 \div 4 = 2 \]

ex: \[ 15a^2 + 20ab - 10ac = ? \]
Solution:

\[ 15a^2 + 20ab - 10ac = 5a(3a + 4b - 2c) \]

note: put variables in alphabetical order or decreasing order of powers

ex Fact: \[ 72x + 36x^2 = ? \]
Solution:

\[ 36x^2 + 72x = 36x(1x + 2) \]
GCF = 36x
Note: 
\[36x^2 + 72x = 9x(4x + 8)\]
\[= 9x(4)(x + 2)\]
\[= 36x(x + 2)\]

\[36x^2 + 72x = 18x(2x + 4)\]
\[= 18x(2)(x + 2)\]
\[= 36x(x + 2)\]

Ex. \[4 - 6x, \text{ is } ?\]

\[\text{Solln.}\]
\[-6x + 4 = -2(3x - 2)\]

\[\text{leading term should be positive.}\]

II Difference of Squares

A quadratic polynomial (i.e. \(x^2\)) with no linear \((x)\) term and a negative constant

Ex. \(4x^2 - 1, 25y^2 - 9, 100y^2 - 49z^2\)

"All differences of squares factor into conjugates" or binomials with different signs

Ex. \(4x^2 - 1 = (2x + 1)(2x - 1)\)
ex \( 25y^2 - 9 = (5y + 3)(5y - 3) \)

ex \( 100y^2 - 49z^2 = (10y + 7z)(10y - 7z) \)

Notes: sometimes you need a common factor first.

ex \( 2x^2 - 50 = (\ ? ) (\ ? ) \)

\[ \begin{align*}
\text{GCF:} & \quad 2 \\
\text{Factor:} & \quad 2(x^2 - 25) \\
& \quad 2(x + 5)(x - 5)
\end{align*} \]

ex Factor and reduce the fraction

\[ \frac{3x + 6}{x^2 - 4} = ? \]

Solved

\[ \frac{3(x+2)}{(x+2)(x-2)} \rightarrow \frac{3}{x-2} \]

III Inspection

- a trinomial, usually a quadratic \((x^2)\) starts with

ex \( x^2 + 4x + 3, \quad x^2 - x - 20, \quad x^2 + 7x + 12 \)

Key: - find 2 number that multiply to the last term and add to the middle term
ex \( x^2 + 4x + 3 \) = \((x + 3)(x + 1)\)
\uparrow \quad \uparrow
\text{add} \quad \text{mult}
3, 1

ex \( x^2 + x - 20 \) = \((x + 4)(x - 5)\)
\uparrow \quad \uparrow
\text{add} \quad \text{mult to -20}
to -1
4, -5

ex \( x^2 + 7x + 12 \) = \((x + 3)(x + 4)\)
\[
\begin{array}{c}
\frac{3 + 4}{3 + 4}
\end{array}
\]

Note: You may need to take a common factor first.

ex \( 3x^2 + 3x - 6 \) = ? \( 1, -2 \) or \((-1, 2)\)
\[
3x^2 + 3x - 6 = 3 \left( x^2 + 1x - 2 \right) = 3 \left( x - 1 \right) \left( x + 2 \right)
\]

ex Simplify
\[
\frac{x^2 + 5x + 6}{x^2 - 9} = ?
\]
\[
\text{Sol:} \quad \frac{(x + 2)(x + 3)}{(x + 3)(x - 3)}
\]
\[
= \frac{x + 2}{x - 3}
\]
IV. Decomposition (aka Product-Sum)

This is used for a quadratic trinomial where the first term is not $1x^2$

$2x^2 + 5x + 2, \; 3x^2 - 2x - 1, \; 6x^2 + 5x - 6$

ex $2x^2 + 7x + 3$

1. Multiply the first and last terms, middle term is the sum

2. Find 2 #’s that give the product and sum

3. Replace the middle term with these #’s (and include the x)

4. Place 2 sets of products $(2x^2 + 6x) + (1x + 3)$ separated by a “+”

5. Find common factors $2x(x + 3) + 1(x + 3)$

\[ \therefore (x + 3)(2x + 1) \]
ex \[ 2x^2 + 5x + 2 \]

\[ \text{sum} \]

\[ \text{prod} = 4 = \frac{4}{1} \cdot \frac{1}{1} \]

\[ \text{sum} = 5 = \frac{4}{1} + \frac{1}{1} \]

\[ \therefore (2x^2 + 4x) + (1x + 2) \]

\[ = 2x(1x + 2) + 1(1x + 2) \]

\[ = (1x + 2)(2x + 1) \]

ex \[ 6x^2 + 5x - 6 \]

\[ \text{sum} \]

\[ \text{prod} = -36 = \frac{9}{9} \cdot \frac{-4}{-4} \]

\[ \text{sum} = 5 = \frac{9}{9} + \frac{-4}{-4} \]

\[ \therefore 6x^2 + 9x = 4x - 6 \]

\[ = (6x^2 + 9x) + (-4x - 6) \]

\[ = 3x(2x + 3) - 2(2x + 3) \]

\[ = (2x + 3)(3x - 2) \]

"Box Method" \[ 6x^2 + 5x - 6 \]

\[ \text{prod} = -36 = \frac{9x - 4}{9 + 4} \]

\[ \text{sum} = 5 = \frac{9}{9} + \frac{-4}{-4} \]

\[ \therefore (2x + 3)(3x - 2) \]
Factor completely

\[
\begin{align*}
&x^2 + 10x = \\
&x^2 - 6x = \\
&2x + 6 = \\
&3x^2 - 15x = \\
&x^2 - 121 = \\
&x^2 - 64 = \\
&4x^2 - 625 = \\
&9x^2 - 49y^2 = \\
&10x^2 - 40 = \\
&8x^2 - 98 = \\
&x^2 + 32x + 60 = \\
&x^2 - 16x + 60 = \\
&x^2 + 61x + 60 = \\
&x^2 + 23x + 60 = \\
&x^2 - 59x - 60 = \\
&x^2 + 11x - 60 = \\
&x^2 - 7x - 60 = \\
&x^2 + 28x - 60 = \\
&4x^2 + 7x + 3 = \\
&2x^2 + 3x - 2 = 
\end{align*}
\]