Standards for Logical Reasoning

General Outcome

Develop logical reasoning.

General Notes:

• This topic is intended to develop students’ numerical and logical reasoning skills, which are applicable to many everyday situations.

Specific Outcome 1

Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]

Notes:

• This is an extension of Mathematics 20–2 Number and Logic Specific Outcome 2, which involves spatial reasoning.

• The intent of this outcome is for students to explore the games and develop personal strategies, with an emphasis on the development of logical reasoning skills.

• A sample list of puzzles and games that could be used in working through this outcome is shown below. These games and puzzles could be integrated throughout the course as an alternative to devoting a fixed number of lessons to them.

<table>
<thead>
<tr>
<th>Logic Puzzles/Games</th>
<th>Strategic Puzzles/Games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tic Tac Chec</td>
<td>Chess</td>
</tr>
<tr>
<td>Mastermind</td>
<td>Cribbage</td>
</tr>
<tr>
<td>Sudoku</td>
<td>Nim</td>
</tr>
<tr>
<td>Kakuro</td>
<td>Battleship</td>
</tr>
<tr>
<td>Logic puzzles</td>
<td>Backgammon</td>
</tr>
<tr>
<td>Magic squares</td>
<td>Othello</td>
</tr>
<tr>
<td>Rush Hour</td>
<td>Sequence</td>
</tr>
<tr>
<td>Strimko</td>
<td>Blokus</td>
</tr>
</tbody>
</table>

• Questions will address the logic and reasoning processes within games or puzzles. Detailed knowledge of specific games will not be assessed on the diploma examination.

• The calendar in each issue of Mathematics Teacher may be a source of puzzle questions for class discussion or small group work.

(Refer to examples 1, 2, 3, 4, 5, 6, 7, and 8)
Specific Outcome 2
Solve problems that involve the application of set theory. [CN, PS, R, V] [ICT: C6-2.3]

Notes:
- Students should be familiar with different classifications of numbers (e.g., number systems, prime and composite numbers, multiples of numbers, factors, etc.).
- Students should be familiar with verbal descriptions, symbols, and graphic organizers when describing sets and applying set theory.
- Teachers should be aware that Venn diagrams and logical reasoning symbols are also used in probability.
- Teachers should be aware that there are other set theory symbols that may be used; however, students should be familiar with the symbols on the formula sheet.
- In set theory, the word “or” is inclusive (i.e., it means “and/or”).

(Refer to examples 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, and 28)
Acceptable Standard
The student can
• solve pattern problems involving logic
• completely solve a puzzle involving logic
• develop a successful strategy within the rules of a game
• integrate additional information to adapt a strategy
• identify errors in a solution to a puzzle or in a strategy for winning a game
• create a variation on a puzzle or a game and develop an appropriate strategy
• use set notation correctly, including the logical reasoning symbols $A'$, $\emptyset$, $\cap$, $\subset$, and $\cup$
• correctly describe complements of sets, the empty set, disjoint sets, subsets, and universal sets in context
• solve problems that involve the analysis of two sets within a universal set
• explain the reasoning used in solving set theory problems that involve two sets
• identify errors in a solution to a problem that involves sets
• participate in and contribute toward the problem-solving process for problems that require the application of logical reasoning studied in Mathematics 30–2

Standard of Excellence
The student can also
• describe a successful strategy within the rules of a game and explain why the strategy works
• identify and correct errors in a solution to a puzzle or in a strategy for winning a game
• justify a change in strategy for solving a puzzle or for winning a game when variations have been made to the puzzle or game
• correctly describe the complement of the result of an operation on two sets. (e.g., $(A \cup B)'$)
• solve problems that involve the analysis of three sets within a universal set
• explain the reasoning used in solving set theory problems that involve three sets
• identify and correct errors in a solution to a problem that involves sets
• complete the solution to problems that require the application of logical reasoning studied in Mathematics 30–2
Sample Questions

Students who achieve the acceptable standard should be able to answer all of the following questions, except for any part of a question labelled SE. Parts labelled SE are appropriate examples for students who achieve the standard of excellence.

Note: In the multiple-choice questions that follow, * indicates the correct answer. Please be aware that the worked solutions shown are possible strategies; there may be other strategies that could be used.

Use the following information to answer questions 1 and 2.

Three rows of a pattern are shown below.

<table>
<thead>
<tr>
<th>Row</th>
<th>1 × 8 + 1 = 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 2</td>
<td>12 × 8 + 2 = 98</td>
</tr>
<tr>
<td>Row 3</td>
<td>123 × 8 + 3 = 987</td>
</tr>
</tbody>
</table>

1. Row 5 of the pattern will be
   A. 1 234 × 8 + 4 = 9 876
   B. 1 234 × 8 + 5 = 9 876
   C. 12 345 × 8 + 4 = 9 876
   *D. 12 345 × 8 + 5 = 98 765

2. If the number 8 in the pattern above is replaced by the number 9 as shown below, describe a pattern that could be used to calculate the value of row 7.

<table>
<thead>
<tr>
<th>Row</th>
<th>1 × 9 + 1 = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 2</td>
<td>12 × 9 + 2 = 110</td>
</tr>
<tr>
<td>Row 3</td>
<td>123 × 9 + 3 = 1 110</td>
</tr>
</tbody>
</table>

   **Possible Solution:**
   The row number tells you the number of ones that occur in the answer followed by a single 0. Therefore, the value of row 7 would be seven ones followed by a single 0 (i.e., 11 111 110). Alternatively, the value of row 7 could be calculated by recognizing the pattern in the operations.
Use the following information to answer question 3.

The goal of a particular puzzle is to fill the circles in a grid with the letters A, B, C, and D so that no letters are repeated in any row, column, or set of connected circles.

The three entries in the grey circles were given to start the puzzle. Jerome has already completed three entries shown in the white circles, but he has made an error.

3. a. Identify the error that Jerome made in his solution to the puzzle.

Possible Solution:
   a. The error that Jerome made is in row 3, column 3.

b. Explain why this entry is incorrect.

Possible Solution:
   b. If Jerome’s entries were correct, then the entry in row 3, column 1 must be an A, and the entry in row 3, column 4 must be a B. However, since the last three circles in row 4 are connected to the circle in row 3, column 4, this will mean that one of the rules of the game is violated because a set of connected circles has repeated letters (i.e., two B’s).
c. Correct the error that Jerome made and complete the puzzle.

Solution:
Use the following information to answer numerical-response question 4.

The goal of a particular two-player game is to create a line of four adjacent rectangles using the same letter. To play, each player takes turns placing their first initial somewhere on a six-by-six grid. Margaret and Gerda have started playing this game, as shown on the grid below.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Row 2</td>
<td></td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Row 3</td>
<td></td>
<td>G</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Row 4</td>
<td>M</td>
<td>M</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Row 5</td>
<td></td>
<td>M</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Row 6</td>
<td>M</td>
<td>M</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Numerical Response

4. It is Margaret’s turn, and she determines that she can guarantee a win by placing the letter M in the rectangle at

Row  ____________
Column ____________

Solution:
42

Note: If Margaret places her next M in Row 4, Column 2, she will have two sets of three M’s available, giving her two options for winning when placing her next M. Gerda can block only one of them and cannot create a line of four adjacent G’s on her next turn, so Margaret is guaranteed to win.

Note: This question could be adapted for use on a digital test. Please consult the site https://questaplus.alberta.ca/ for more examples of this item type.
Use the following information to answer question 5.

Four girls – Alice, Brenda, Cathy, Dianna – attended a recreation centre in a large city. Each girl participated in a different activity – Weight Training, Skating, Jogging, Swimming – and each girl owns a backpack of a different colour – Blue, Pink, Green, Red. The four clues below provide information about the activity in which each girl participated and the colour of each girl’s backpack.

- Cathy does not have a red backpack and needed to dress warmly for her activity.
- The swimmer who owned the red backpack was not Brenda.
- The girl who carried the green backpack was jogging.
- While preparing for her jog, Dianna watched her friend take her skates out of her blue backpack.

<table>
<thead>
<tr>
<th>Weight Training</th>
<th>Skating</th>
<th>Jogging</th>
<th>Swimming</th>
<th>Blue</th>
<th>Pink</th>
<th>Green</th>
<th>Red</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brenda</td>
<td>✓✓</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cathy</td>
<td>×</td>
<td>✓</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dianna</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Determine the activity and backpack colour for each girl.

Possible Solution:

<table>
<thead>
<tr>
<th>Weight Training</th>
<th>Skating</th>
<th>Jogging</th>
<th>Swimming</th>
<th>Blue</th>
<th>Pink</th>
<th>Green</th>
<th>Red</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brenda</td>
<td>✓✓</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cathy</td>
<td>×</td>
<td>✓</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dianna</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>×</td>
<td>✓</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pink</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>
Use the following information to answer question 6.

A pattern of pictures is shown below. In Step 2, each shaded square has stayed in the same place as in Step 1 or moved to a square horizontally, vertically, or diagonally adjacent to its location in Step 1. The shaded square undergoes the same movement in each subsequent step.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Step 1" /></td>
<td><img src="image" alt="Step 2" /></td>
<td><img src="image" alt="Step 3" /></td>
</tr>
</tbody>
</table>

6. Which of the following pictures is next in the pattern?

* A.  

* B.  

* C.  

* D.  

Possible Solution:
The shaded box in the first column is moving upwards in each step. The shaded boxes in the second and third columns are remaining stationary. The remaining shaded box, which started in Column 4, moves diagonally down and to the left.
Use the following information to answer numerical-response question 7.

A student makes the following statement.

“VOTE compares to VETO as the number 8570 compares to the number __________.”

**Numerical Response**

7. The 4-digit number that completes the statement above is __________.

**Solution:**

8075
Use the following information to answer question 8.

A geometric pattern is presented below. In this pattern, the lower left square is unknown.

8. Which of the following pictures could be in the lower left square to complete the geometric pattern shown above?

A. ![A](image1)
B. ![B](image2)
*C.* ![C](image3)
D. ![D](image4)

Possible Solution:
A comparison of the squares in the top row reveals that
• the direction of the slanted dividing line changes, but the same shapes are preserved. The blank square must therefore be a square shape within the square shape having a slanted dividing line that goes down and to the right.
• the colours of the inner triangles are the same but their positions are switched. The inner triangle of the blank square must therefore have a pink exterior and a green interior.
• in the outer triangles, yellow has changed to grey, and red has changed to black. The outer triangle of the blank square must therefore have a black exterior and a grey interior.
Use the following information to answer questions 9 and 12 and numerical-response questions 10 and 11.

Students in a particular high school were surveyed to determine the courses in which they were currently enrolled. The table below represents the data that were collected.

<table>
<thead>
<tr>
<th>Courses Enrolled</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math only</td>
<td>28</td>
</tr>
<tr>
<td>Art only</td>
<td>33</td>
</tr>
<tr>
<td>Math and Art</td>
<td>17</td>
</tr>
<tr>
<td>Neither course</td>
<td>20</td>
</tr>
</tbody>
</table>

9. The number of students in the universal set is

   A. 61
   B. 64
   C. 78
   *D. 98

Numerical Response

10. The number of students taking Art is __________.

   Solution:
   50

   There are 33 who take Art only and 17 who take both Math and Art.
Numerical Response

11. The number of students not taking Math is __________.

Solution:
53

There are 33 who take Art only and 20 who take neither.

12. The number of students taking Math or Art is

A. 17
B. 61
*C. 78
D. 98

Use the following information to answer question 13.

There are 35 students in John’s homeroom class. There are 5 students who take English and biology, and 7 students who take neither of these subjects. There are 3 more students taking English only than there are students taking biology only.

13. The number of students in John’s homeroom who take biology only is

*A. 10
B. 13
C. 15
D. 20
Use the following information to answer questions 14, 16, and 17 and numerical-response question 15.

A group of 100 students was surveyed about movies that they have seen, as shown below.

- 2 people saw all three movies
- 12 people saw “Metal Man” and “The Princely Groom”
- 53 people saw “Metal Man”
- 10 people saw “Metal Man” and “Quick and Angry”
- 18 people saw “The Princely Groom” only
- 23 people saw “The Princely Groom” and “Quick and Angry”
- 6 people did not see any of the movies

Jason started to organize the results in the Venn diagram shown below.
14. The number of students who saw “The Princely Groom” is
   A. 18
   B. 20
   *C. 51
   D. 53

**Numerical Response**

15. The number of students who saw “Metal Man” and “The Princely Groom” but not “Quick and Angry” is __________.

   **Solution:**
   10

16. The number of students who saw “Metal Man” only is
   A. 20
   *B. 33
   C. 51
   D. 53

17. The number of students who saw “Metal Man” or “Quick and Angry” is
   A. 10
   B. 43
   *C. 76
   D. 98
Use the following information to answer questions 18 and 19.

**Two Sets**

\[ A = \{ \text{prime numbers less than 20} \} \]
\[ B = \{ \text{factors of 20} \} \]

18. The union of sets \( A \) and \( B \) is

A. \( \{2, 5\} \)
B. \( \{2, 4, 5, 10\} \)
C. \( \{1, 2, 3, 4, 5, 7, 10, 11, 13, 17, 19, 20\} \)
D. \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\} \)

19. The set representing \( A \cap B \) is

A. \( \{2, 5\} \)
B. \( \{2, 4, 5, 10\} \)
C. \( \{1, 2, 3, 4, 5, 7, 10, 11, 13, 17, 19, 20\} \)
D. \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\} \)
Use the following information to answer numerical-response question 20.

### Four Sets

<table>
<thead>
<tr>
<th>Set</th>
<th>Set Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{2, 3, 8, 9, 15, 16}</td>
</tr>
<tr>
<td>2</td>
<td>{2, 3, 5, 7, 11, 13, 17, 19}</td>
</tr>
<tr>
<td>3</td>
<td>{multiples of 3 between 0 and 20}</td>
</tr>
<tr>
<td>4</td>
<td>{odd numbers between 0 and 20}</td>
</tr>
</tbody>
</table>

**Numerical Response**

20. A pair of sets that will produce the intersection \{3, 9, 15\} are _____ and _____.
   (There is more than one correct answer.)

**Possible Solutions:**
13, 31, 14, 41, 34, 43
21. Which of the following rows includes two groups that are disjoint sets?

<table>
<thead>
<tr>
<th>Row</th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>People who drink coffee</td>
<td>People who do not drink tea</td>
</tr>
<tr>
<td>B.</td>
<td>People who have a home phone line</td>
<td>People who have a cellular phone line</td>
</tr>
<tr>
<td>C.</td>
<td>People who are left-handed</td>
<td>People who own a computer</td>
</tr>
<tr>
<td>D.</td>
<td>People who live in Calgary</td>
<td>People who do not live in Alberta</td>
</tr>
</tbody>
</table>

Use the following information to answer question 22.

Vehicles with a sunroof are represented by $S$ and vehicles with a hands-free phone system are represented by $P$.

22. Which of the following vehicle descriptions illustrates $(S \cap P)'$?

A. A sunroof or a hands-free phone system
B. A sunroof and not a hands-free phone system
*C. Not a sunroof or not a hands-free phone system
D. Not a sunroof and not a hands-free phone system
Use the following information to answer question 23.

In a school of 120 students:

- 15 students took Physics and English
- 45 students took English
- 30 students took Physics

Bobby incorrectly summarized the data using the Venn diagram shown below.

![Venn Diagram](image)

23. Identify the regions of Bobby’s Venn diagram that have incorrect entries, and describe the errors that Bobby made. Make changes to the Venn diagram to show the correct entries.
**Possible Solution:**
When completing the Venn diagram, Bobby made errors in the regions that represent the number of students taking Physics only and the number of students taking English only. The number of students taking both courses is 15 and these students are already counted in the students taking Physics and the students taking English. Consequently, the number of students taking Physics only should be 30 – 15 or 15 students. The number of students taking English only should be 45 – 15 or 30 students. This will make the region within the universal set, but outside the circles, become 120 – (15 + 15 + 30) or 60 students. The correct Venn diagram is shown below.
Use the following information to answer questions 24 and 25.

Three Sets

\[ R = \{ \text{natural numbers less than 50} \} \]
\[ S = \{ \text{even numbers} \} \]
\[ T = \{10, 20, 30, 40\} \]

24. Which of the following statements is true for sets \( R \), \( S \), and \( T \)?

A. \( R \subset S \)
B. \( R \subset T \)
C. \( S \subset R \)
*D. \( T \subset R \)

25. Which of the following statements is not true for sets \( R \), \( S \), and \( T \)?

A. \( T \subset (R \cap S) \)
B. \( T \subset (R \cap T) \)
*C. \( (R \cap S) \subset T \)
D. \( (R \cap T) \subset T \)
26. Is this student correct or incorrect? Use an example or a visual representation in your explanation.

**Possible Solution:**
The student is correct that $A \cup \emptyset = A$. No elements are added to $A$ by combining it with the empty set, so the initial set $A$ remains after the union. For example, if $A = \{2, 3, 4\}$, and it is combined with a set that has no elements, the result will be $\{2, 3, 4\}$. The student is incorrect that $A \cap \emptyset = A$. Since the empty set has no elements, it cannot have any elements in common with $A$, so $A \cap \emptyset = \emptyset$. For example, if $A = \{2, 3, 4\}$, none of these elements occurs in $\emptyset$; there are no common elements, meaning the set of common elements is $\emptyset$.

27. Which of the following Venn diagrams illustrates $M \cap N = \emptyset$ for all sets $M$ and $N$?

A. 

```
M
```

```
N
```

B. 

```
N
```

```
M
```

*C. 

```
M
```

```
N
```

D. 

```
M
```

```
N
```

28. Which of the following phrases describes an empty set?

A. Common factors of 3 and 7
B. Prime numbers that are even
C. Multiples of 5 that are less than 10
*D. Perfect squares less than 20 that are divisible by 5