Sessions

re integers, and \( b \neq 0 \).

1. Determine the restrictions on the values of \( x \).
   \[
   \frac{12}{x + 9} \quad \frac{x + 12}{x^2 + 9x + 14} \quad \frac{3x^2 - 2x - 5}{2x^2 - 9x + 9} \quad \frac{8x}{6x - 16}
   \]

2. Simplify using common factors.
   \[
   \frac{5ab^3}{-16b} \quad \frac{27mn^p}{63mn^q} \quad \frac{105p^q - 18pq}{25p^2 - 4q^2}
   \]

   \[
   \frac{3x^2 - 6x + 15x}{4x + 12} \quad \frac{15ab + 45ac}{20a^2 + 20ac} \quad \frac{16m^n - 32mn^2}{72m^n - 48n^2}
   \]

4. Express each in lowest terms.
   \[
   \frac{x^2 + x - 6}{x - 3} \quad \frac{x^2 + 2x - 24}{x + 18} \quad \frac{y(x + 3) - 2(x + 3)}{x^2 + 6x + 9}
   \]

5. Simplify. Identify the restrictions on the variables.
   \[
   \frac{2x^2 - x - 15}{x^2 + x - 12} \quad \frac{10x^2 - 3x - 4}{x^2 - 15x + 16} \quad \frac{3x^2 + 13x - 10}{9x^2 - 4}
   \]

6. Express each in lowest terms.
   \[
   \frac{2a^2 + 5ab - 3b^2}{a^2 - 9b^2} \quad \frac{12x^2 + 28xy - 5y^2}{6x^2 + 5xy - y^2} \quad \frac{9a^2 + 12ab + 4b^2}{3a^2 - 16ab - 12b^2}
   \]

7. Determine the restrictions on \( x \) and simplify.
   \[
   \frac{5x^2 + 13x + 6}{x^2 + 5x + 6} \quad \frac{6x^2 - 6x - 20}{x^2 - 9x + 9} \quad \frac{12x^2 + 33x}{x^2 - 9x + 20}
   \]

8. Simplify. Identify the restrictions on the variables.
   \[
   \frac{2x^2 - x - 4}{2x^2 + 16x + 32} \quad \frac{2a^2 + 6ab - 36b^2}{10a^2 - 28ab - 6b^2}
   \]

9. A farmer divides his herd of cattle among his three children. The eldest is given half the herd, the youngest receives one sixth, and the middle child receives 6 more than the youngest. How large was the farmer’s herd?

10. Calculate the average speed of a 24 km round trip if half of the trip was covered at 8 km/h and the other half at 12 km/h.

Historical Note

Johann Kepler (1571-1630)

It has been said that almost any problem can be solved if one continuously worries over it and works on it for a sufficiently long time. Thomas Edison once said that inventions are one percent inspiration and ninety-nine percent perspiration. This is demonstrated by Johann Kepler’s tenacity in solving problems concerning planetary motion. He was convinced that planets revolved in orbits around a central sun, and he wanted to determine the motion and position of these orbits. Kepler used Tycho Brahe’s observations on the motion of planets. The problem was to obtain a pattern of motion that coincides with Brahe’s observations. Kepler arrived at a hypothesis and then sifted through a “mountain” of tedious calculations to confirm or reject the hypothesis. After twenty-one years, he succeeded in formulating his first two laws of planetary motion, and ten years later, he formulated his third law. Using the library, research and explain Kepler’s three laws.
Rational

Exercises

1. Multiply and simplify.
   a. \( \frac{5}{25} \times \frac{10}{9} \)
   b. \( \frac{7}{5} \times \frac{20}{35} \times \frac{21}{24} \)
   c. \( \frac{15a}{6b} \times \frac{4ab}{10a} \)
   d. \( \frac{10 \times 8y}{2x} \times \frac{-6}{8y} \times \frac{4y}{10x} \)
   e. \( \frac{24m^2}{15n^3} \times \frac{8nm}{12m^3} \)
   f. \( \frac{-18a^2b}{10a} \times \frac{15b}{20ab^2} \)

2. Divide and simplify.
   a. \( \frac{a}{b} \div \frac{3}{8} \)
   b. \( \frac{b}{a} \div \frac{5}{16} \)
   c. \( \frac{8x^2}{15a^2} \div \frac{12a^2}{25x} \)
   d. \( \frac{3a^2}{15b^2} \div \frac{10a}{16b} \)
   e. \( \frac{40m^2n}{9m^2} \div \frac{24m^4}{56a^3} \)
   f. \( \frac{56a^3}{16a^2b} \div \frac{30ab^2}{15ab} \)

3. Perform the operations and simplify.
   a. \( \frac{12a}{20b^2} \times \frac{15ab}{24} + \frac{30a^2b}{36b^3} \)
   b. \( \frac{4x}{9y} \div \frac{10xy}{x^2yz} \)
   c. \( 6 \times \frac{3a}{5b} + \frac{18a}{10b} \)
   d. \( \frac{4x}{3y} \times \frac{30x^2}{12y} \times \frac{48y^2}{54x + 5x^2} \)

4. Multiply and simplify.
   a. \( \frac{x^2 - 25}{8x} \)
   b. \( \frac{x^2 + 3x - 18}{x^2 - x - 6} \)
   c. \( \frac{2x^2 - 32}{x + 6} \)
   d. \( \frac{20x^3}{4x^2 - 36x} x^2 + 2x - 8 \)
   e. \( \frac{25x^3}{x^2 - 25} \)

5. An engineer designed a triangular steel structure with these dimensions.
   a. What is its area?
   b. If \(a = 21\) cm and \(b = 8\) cm, then what is its actual area?

6. Perform the operations and simplify. Determine the restrictions on the variable.
   a. \( \frac{a^2 - 3a - 10}{a - 5} \div \frac{3a^2 + 7a - 6}{a + 3} \)
   b. \( \frac{3x^2 - 12x^2}{x^2 - 8 + 16} \times \frac{x - 4}{3x^2} \)

7. Simplify by performing the indicated operations.
   a. \( 6x^2 - 7x - 5 \times 8x^2 - 18x - 5 \)
   b. \( 4x^2 - 8x - 5 \times -7x - 3 \)
   c. \( 4x^2 + 20 \div \frac{x^2 - 25}{6x^2 - 6} \div \frac{5x^2 + 5x}{5x} \)
   d. \( \frac{8x - 5}{12x^2} \times \frac{8x^2 + 11x - 10}{x + 2} \)
   e. \( 15x^2 + x - 2 \times \frac{4x^2 + 17x + 15}{12x^2 + 11x - 5} \)
   f. \( \frac{4x - 12x + 9}{4x - 12x + 9} \times \frac{18x - 12x^2}{3x + 16x + 5} \)
   g. \( 2x^2 + ab - 15b^2 \times \frac{8a^2 + 4ab}{4a^3 - 8ab - 5b^2} \)

8. Determine the restrictions and perform the indicated operations to simplify.
   a. \( \frac{6x^2 + 12x}{x^2 - 3x - 10} \div \frac{x^2 - 25}{4x} \)
   b. \( \frac{6x^2 - 23x - 4}{x + 2} \div \frac{6x^2 - 17x - 3}{x + 3} \)
   c. \( \frac{x^2 - 4}{2 - x} \)

9. A triangle has a height of \( \frac{2x + 3}{x + 1} \) units and an area of \( \frac{x^2 + 4x + 3}{2x^2 + 7x + 3} \) square units. Find the base length of the triangle in simplest form.

10. If the total distance travelled from Regina to Saskatoon is given by the expression \( \frac{3x^2 + x - 4}{2x - 4} \) km and the rate of travel is given by the expression \( \frac{3x^2 + 4}{x^2 - 4} \) km/h, then find an expression for the total time required to travel the total distance.

Using the Library

Write a short essay on continuing fractions. Through the use of examples, explain how they are related to irrational numbers.
Exercises

1. Find the LCM of each.
   a. 16, 24  b. 18x, 30xy  c. 12a^2, 20ab  d. 9x^2y, 27xy^2
   e. 15x, 5xy, 20y  f. 2x^2, 6xy, 8xy^2  g. 6ab, 3bc, 12ac
   h. 3x^2, 5y, 6xy

2. Write each expression with a denominator of 5xy.
   a. 3x  b. \( \frac{2x}{y} \)
   c. \( \frac{3 + x + y}{2} \)  d. \( \frac{2x - 8y}{2} \)

3. Add. Write your answer in lowest terms.
   a. \( \frac{6x}{5} + \frac{3y}{2} \)  b. \( \frac{3a}{4} + \frac{a}{8} \)
   c. \( \frac{5x}{12} + \frac{7x}{18} \)  d. \( \frac{x}{6} + \frac{3x}{10} \)

4. Subtract. Write your answer in lowest terms.
   a. \( \frac{10a}{3} - \frac{5b}{2} \)  b. \( \frac{4x}{9} - \frac{x}{6} \)
   c. \( \frac{2x}{15} - \frac{9x}{10} \)  d. \( \frac{15x}{8} - \frac{3}{2} \)

5. Write each as an equivalent rational expression with the denominator indicated.
   a. \( \frac{5x}{y^2} \)  b. \( \frac{3x + 1}{x} \)
   c. \( \frac{4x}{y} \)  d. \( \frac{x + 1}{y(y - 1)} \)
   e. \( \frac{2y + 2}{x} = \frac{2(x + 1)}{3(x - 1)} \)  f. \( \frac{2(x + 1)}{3(x - 1)} = \frac{x}{y(x - 1)} \)

6. Perform the indicated operations and write your answer in lowest terms.
   a. \( \frac{5a}{2a} + \frac{10}{3a^2} \)  b. \( \frac{3}{4x^2} - \frac{7}{10x} \)
   c. \( \frac{2}{9mn^2} - \frac{5}{12m^n} \)  d. \( \frac{2c + 3b}{ac} \)
   e. \( \frac{3}{4x} + \frac{5}{6y} - \frac{9}{8z} \)  f. \( \frac{10}{a} - \frac{3c}{ab} + \frac{2}{c} \)

7. Simplify.
   a. \( \frac{2x + 5}{3} + \frac{x - 1}{4} \)  b. \( \frac{16a}{5} - \frac{a + 2}{2} \)
   c. \( \frac{5x - 6}{12} + \frac{x + 4}{3} \)  d. \( \frac{2x + 10}{15} + \frac{3x}{20} \)
   e. \( \frac{a - 3}{5} + \frac{a + 2}{2} + \frac{2a + 1}{4} \)

8. Simplify. Determine the restrictions.
   a. \( \frac{8}{2x + 3} + \frac{x - 5}{2x + 3} \)  b. \( \frac{6x - 2}{x + 4} - \frac{4x + 1}{x + 4} \)
   c. \( \frac{x + 6}{x + 5} + \frac{2x + 9}{x + 5} \)  d. \( \frac{2x^2 + 1}{2x + 1} + \frac{13x + 1}{2x + 1} \)

9. Perform the indicated operations and simplify.
   a. \( \frac{2x + 3}{6x} + \frac{x - 4}{8x^2} \)  b. \( \frac{4a - 2a - 5}{3b} + 2 \)
   c. \( \frac{6x + 1}{5x} + \frac{2x - 12}{10} \)  d. \( \frac{8a - 3b - a - 10}{4ab} - \frac{6a}{6a^2} \)
   e. \( \frac{1}{x^2} + \frac{2y}{3x^2} + \frac{5x}{6y} \)  f. \( \frac{5}{2p} + \frac{3}{5q} - \frac{2}{3pq} \)
   g. \( \frac{2m}{2m} - \frac{m - 3}{3m} + \frac{5mn + 1}{6} \)
   h. \( \frac{9x^2 - 15x}{2} - \frac{7x - 4}{8} \)

10. Add \( \frac{5x^2 - 8x - 3}{x - 3} \) to the sum of \( \frac{3x + 12}{x - 3} \). Write your answer in simplest form.

11. Simplify.
    a. \( \frac{14x - 2}{4x - 6} - \frac{6x + 4}{4x - 3} \)
    b. \( \frac{x^2 - 6x - 10}{2x + 7} + \frac{x^2 + 7x - 11}{2x + 7} \)
    c. \( \frac{x^2 - 12x + 3}{x^2 - 8x - 20} + \frac{2x^2 - 17x - 13}{x^2 - 8x - 20} \)
    d. \( \frac{3x^2 + 7x}{2x^2 + 8} - \frac{x^2 - 3x + 2}{x^2 - 3x + 2} \)
    e. \( \frac{5x^2 + 3x - 4}{2x^2 + 11x + 15} - \frac{4x^2 + 3x + 5}{2x^2 + 11x + 15} \)

12. Perform the indicated operations and simplify. Determine the restrictions on \( x \).
    a. \( \frac{10x}{x + 6} + \frac{2x^2 - 5}{x + 6} - \frac{x + 13}{x + 6} \)
    b. \( \frac{8}{15x} + \frac{4 - 3x}{6x^2} \)

13. A foreman installing a steel pipeline wanted a pipe that is 24 cm in diameter. When he went to the steel plant, he found that there was no 24 cm diameter pipe. However, the plant offered him two steel pipes each with a diameter of 12 cm. The foreman thought that he could combine the two smaller pipes to replace the larger one. Was he correct in his thinking?